

An Automata-Based Approach to Games with ω -Automatic Preferences

Highlights'26 – Vienna

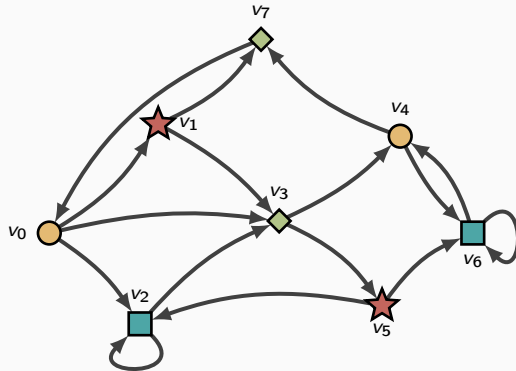
Christophe Grandmont

September 11, 2026



Joint work with Véronique Bruyère, Emmanuel Filiot, Jean-François Raskin

Game played on graphs



Game played on graphs

G

game theory

Games

games on graphs

Games played on graphs

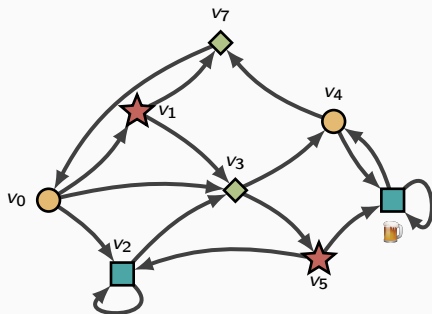
graph dimension

Graph games



Game played on graphs: Nash Equilibria

Does there exist an **Nash Equilibrium** (NE) satisfying some **constraints**?



- NE with **reachability** objectives?

Player ●: reach v_3

Player ■: reach v_5

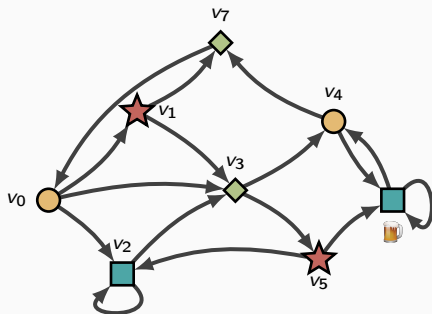
Player ◇: reach v_2

Player ★: reach 🍺

Does there exist an NE visiting 🍺?

Game played on graphs: Nash Equilibria

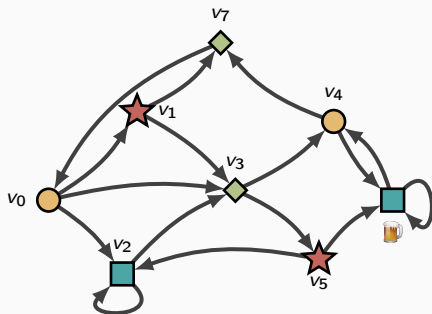
Does there exist an **Nash Equilibrium** (NE) satisfying some **constraints**?



- NE with **reachability** objectives? **Clear**

Game played on graphs: Nash Equilibria

Does there exist an **Nash Equilibrium** (NE) satisfying some **constraints**?



- NE with **reachability** objectives? **Clear**
- NE with **combination of objectives**.

Player $\circ$: some parity objective

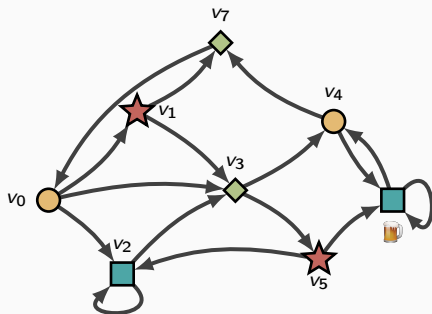
Player $\square$: reach v_3 and v_7 , then reach v_5

Player $\diamond$: some LimSup objective

Player $\star$: Büchi(beer glass)

Game played on graphs: Nash Equilibria

Does there exist an **Nash Equilibrium** (NE) satisfying some **constraints**?



- NE with **reachability** objectives? **Clear**
- NE with **combination of objectives**.

Player : some parity objective

Player : reach v_3 and v_7 , then reach v_5

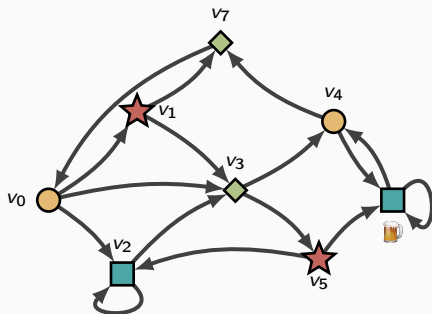
Player : some LimSup objective

Player : Büchi()

Less clear...

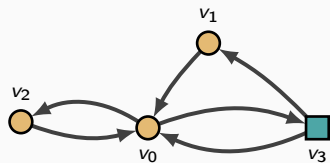
Game played on graphs: Nash Equilibria

Does there exist an **Nash Equilibrium** (NE) satisfying some **constraints**?



There is a **unified framework** where this problem is **PSPACE-complete**.

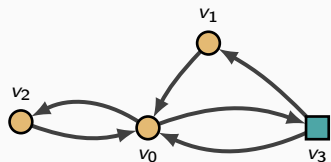
A Unified Framework



Classical setting: **objectives** $\Omega_i : V^\omega \rightarrow \mathbb{Q}$, for each player i ,

e.g., $\Omega_i : V^\omega \rightarrow \{0, 1\}$

A Unified Framework



Classical setting: **objectives** $\Omega_i : V^\omega \rightarrow \mathbb{Q}$, for each player i ,

e.g., $\Omega_i : V^\omega \rightarrow \{0, 1\}$

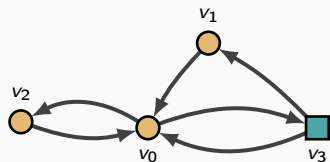
Broader setting: **preference relations**

$\preceq_i \subseteq V^\omega \times V^\omega$ to compare plays:

$\preceq_i \pi \preceq_i \pi'$ if π' is **preferred** to π ,

e.g., $\pi \preceq_i \pi'$ if $\Omega_i(\pi) < \Omega_i(\pi')$.

A Unified Framework



Classical setting: **objectives** $\Omega_i : V^\omega \rightarrow \mathbb{Q}$, for each player i ,

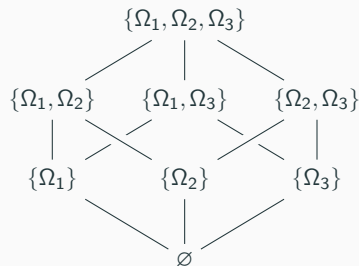
e.g., $\Omega_i : V^\omega \rightarrow \{0, 1\}$

Broader setting: **preference relations**

$\preceq_i \subseteq V^\omega \times V^\omega$ to compare plays:

$\leadsto \pi \preceq_i \pi'$ if π' is **preferred** to π ,

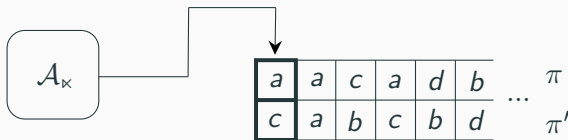
e.g., $\pi \preceq_i \pi'$ if $\Omega_i(\pi) < \Omega_i(\pi')$.



$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.

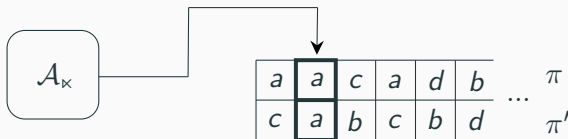
ω -Automatic Relations

$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.



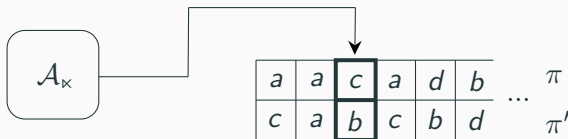
ω -Automatic Relations

$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.



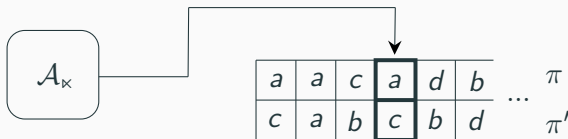
ω -Automatic Relations

$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.



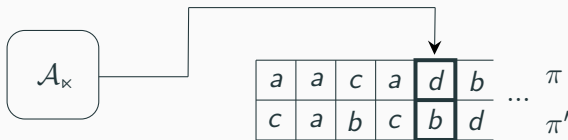
ω -Automatic Relations

$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.



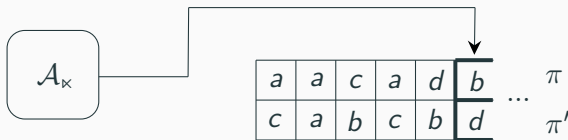
ω -Automatic Relations

$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.



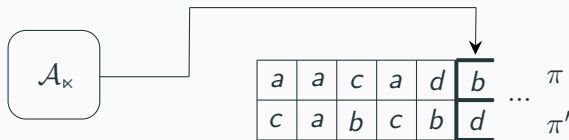
ω -Automatic Relations

$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.



ω -Automatic Relations

$\leadsto$ Define $\times$ with a **deterministic parity automaton** (DPW) $\mathcal{A}_\times$ on the alphabet $V \times V$ that **synchronously** reads two ω -words.



$\mathcal{L}(\mathcal{A}_\times) \subseteq (V \times V)^\omega$ can be seen as a binary relation: **ω -automatic relation!**

Example: $\{(x, y) \mid \text{first}_a(x) \leq \text{first}_a(y)\}$ is ω -automatic

Expressivity – Classical Objectives

$$x \ltimes y \text{ iff } \Omega(x) < \Omega(y)$$

Reach()



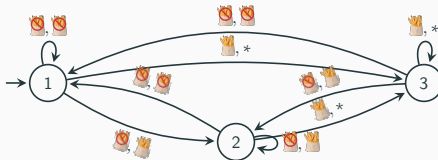
Expressivity – Classical Objectives

$$x \ltimes y \text{ iff } \Omega(x) < \Omega(y)$$

Reach()



Buchi()



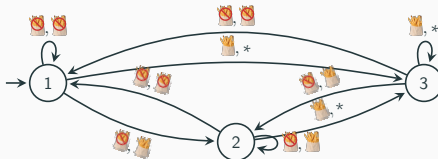
Expressivity – Classical Objectives

$$x \ltimes y \text{ iff } \Omega(x) < \Omega(y)$$

Reach()



Buchi()



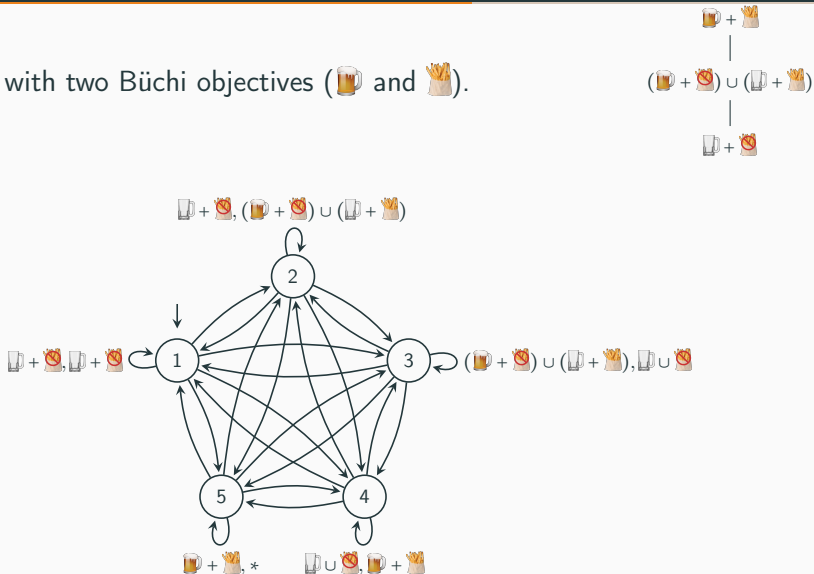
Shortest Path

Reach()



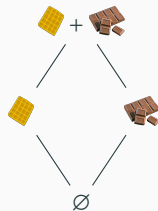
Expressivity – Combinations of Classical Objectives

Counting order with two Büchi objectives (🍺 and 🍟).



Expressivity – Combinations of Classical Objectives

Subset order with two reachability objectives (🟡, 🍫): 12 states.



Rock paper scissors like:



Conjunctions, disjunctions, counting, lexicographic, subset ordering, ...

Threshold problem

Given a lasso $\pi \in \text{Plays}(v_0)$, does player 1 have a **winning strategy for the objective**
 $\Omega = \{\rho \in V^\omega \mid \pi \ltimes \rho\}$?

Zero-Sum Games

Threshold problem

Given a lasso $\pi \in \text{Plays}(v_0)$, does player 1 have a **winning strategy for the objective** $\Omega = \{\rho \in V^\omega \mid \pi \ltimes \rho\}$?

Study values: $\text{Val}_1(v) = \{\pi \in \text{Plays}(v) \mid \exists \sigma_1 \forall \rho \in \text{Plays}_{\sigma_1}(v), \pi \ltimes \rho\}$

Theorem

$\text{Val}_1(v)$ is accepted by a polysize **alternating** parity automaton (APW).

Given	Question	Equiv. to	Complexity
π	Threshold prob. with π ?	$\pi \in \text{Val}_1(v_0)$?	Parity-complete
/	$\exists \pi$, threshold prob. with π ?	$\text{Val}_1(v_0) \neq \emptyset$?	PSPACE-complete

Constrained NE existence problem

Does there exist an NE $\bar{\sigma}$ such that $\langle \bar{\sigma} \rangle_{v_0} \in C$ for an ω -regular constraint $C \subseteq V^\omega$ given as an **APW**.

Constrained NE existence problem

Does there exist an NE $\bar{\sigma}$ such that $\langle \bar{\sigma} \rangle_{v_0} \in C$ for an ω -regular constraint $C \subseteq V^\omega$ given as an APW.

$$\leadsto \text{NE}_{\text{out}}(v) = \bigcap_{i \in \mathcal{P}} (\text{Val}_i(v))^c.$$

Theorem

The NE threshold problem is PSPACE-complete.

Decision Problems

Constrained NE existence problem

Does there exist an NE $\bar{\sigma}$ such that $\langle \bar{\sigma} \rangle_{v_0} \in C$ for an ω -regular constraint $C \subseteq V^\omega$ given as an **APW**.

$$\rightsquigarrow \text{NE}_{\text{out}}(v) = \bigcap_{i \in \mathcal{P}} (\text{Val}_i(v))^c.$$

Theorem

The NE threshold problem is PSPACE-complete.

Rational Synthesis (Stackelberg setting with NEs):

The *cooperative Rational Synthesis* is **PSPACE-complete**,

The *non-cooperative Rational Synthesis* is **undecidable** (Σ_1^1 -hard)

↪ Main future work: extend to **subgame perfect equilibria (SPE)**.

Already **EXPTIME-complete** using **ω -recognizable relations** (finitely many payoffs).

↪ Main future work: extend to **subgame perfect equilibria (SPE)**.

Already **EXPTIME-complete** using **ω -recognizable relations** (finitely many payoffs).

Thank you! Questions?