Games with ω-Automatic Preference Relations

50th MFCS, Warsaw, Poland

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Reactive systems

Continuous interactions between multiple **independent agents** with their own interests.

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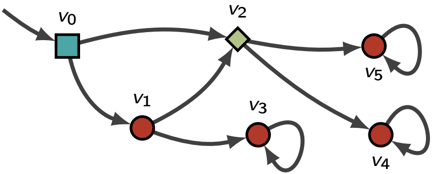
$\rightsquigarrow$ Study this rationality, ensure some specification under rational assumptions.

Game played on graphs

$\leadsto$ How to model these interactions?

Game played on graphs

⇝ Model interactions with **games played on graphs**.



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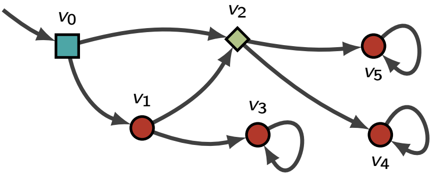

Directed graph: (V, E)
Set of players: $\mathcal{P} = \{1, \ldots, n\}$
Partition of V: $(V_i)_{i \in \mathcal{P}}$

$\left.\right\}$ Arena $A = (V, E, \mathcal{P}, (V_i)_{\in \mathcal{P}})$

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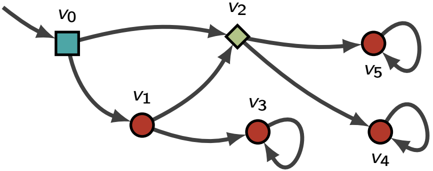

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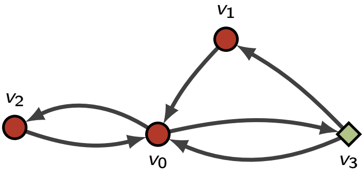
- Play: $\pi \in \text{Plays} \subseteq V^\omega$ consistent with E, history: $h \in V^*$,
- Strategy for $i \in \mathcal{P}$: function $\sigma_i : V^* V_i \to V$, $hv \mapsto \sigma_i(hv)$.

Example - Games played on graphs

Define the following goals:

- player ○ wants to visit v_1 at least once,
- player ◇ wants to visit v_2 infinitely often.



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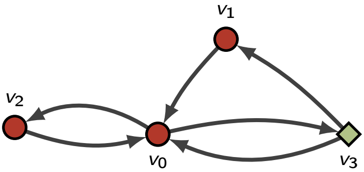

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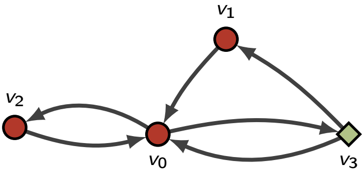


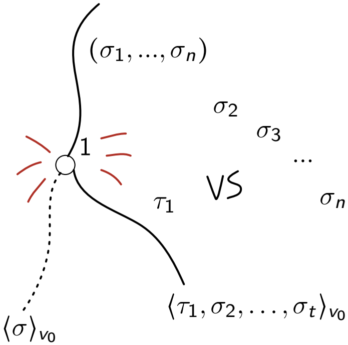
Can players ensure their goals?

No.

Nash Equilibria

A **Nash Equilibrium** (NE) is a strategy profile $\sigma = (\sigma_i)_{i \in \mathcal{P}}$ from which no player has the incentive to **unilaterally deviate**, i.e.,

$$\forall i \in \mathcal{P}, \forall \tau_i \text{ stategy of player } i,\ \langle \sigma_{-i}, \tau_i \rangle_{v_0} \text{ is not better than } \langle \sigma \rangle_{v_0} \textbf{ for } i.$$



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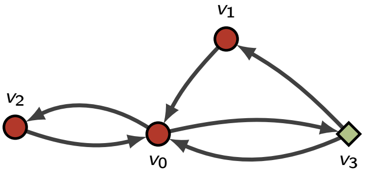

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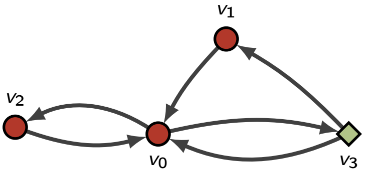

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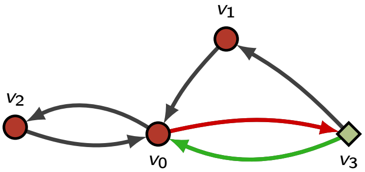

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1) $\sigma_\diamond(hv_3) = v_0$, $\sigma_\circ(hv_0) = v_3$.

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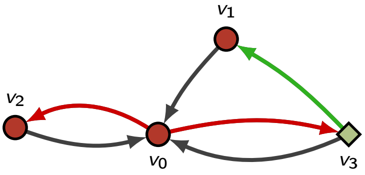

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1) $\sigma_\diamond(hv_3) = v_0$, $\sigma_\circ(hv_0) = v_3$.
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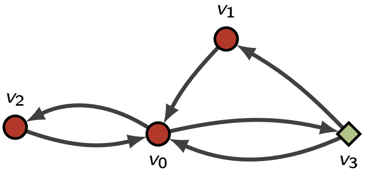

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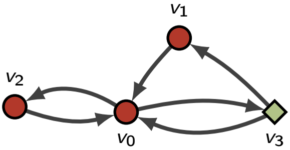
NE 2) is strictly better than NE 1) for both players.

Broader objectives

Classical setting: **objectives** $\Omega_i : \text{Plays} \to \mathbb{Q}$.



What if we take more complex objectives for both players?

Study NEs in all cases?

[1] See, e.g., Bouyer et al., *Nash Equilibria for Reachability Objectives in Multi-player Timed Games*, or Pauly, Le Roux, *Equilibria in multi-player multi-outcome infinite sequential games*.

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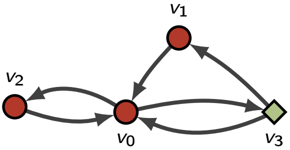

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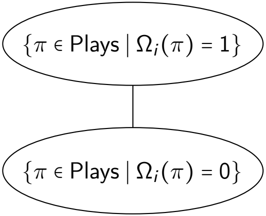
Broader setting: "**preference relations**" $\prec_i \subseteq V^\omega \times V^\omega$ to compare plays.[1]



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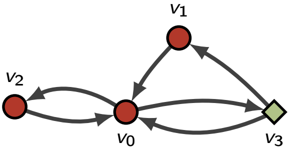

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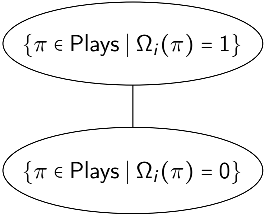

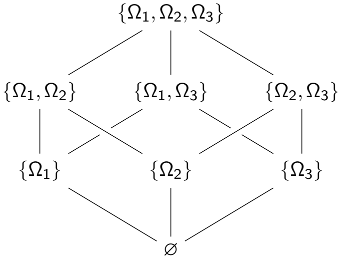

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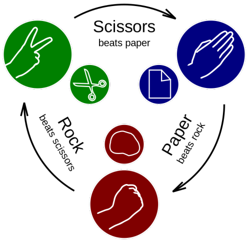
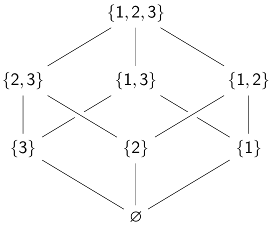
Properties of preference relations





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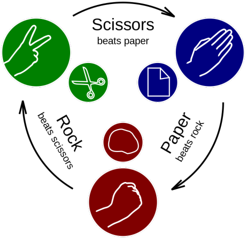

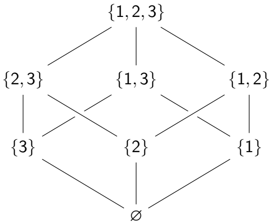

Usual properties

- Reflexivity: $\forall x,\ x \prec x$,
- Irreflexivity: $\forall x,\ x \nprec x$,
- Transitivity: $\forall x, y, z,\ x \prec y \wedge y \prec z \Rightarrow x \prec z$,
- Totality: $\forall x, y,\ x \neq y \Rightarrow x \prec y \vee y \prec x$,
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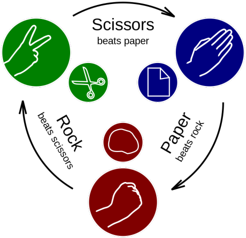

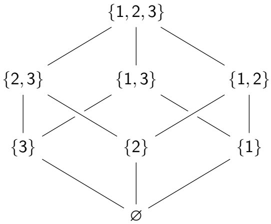

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We expect $\prec$ to have an **order structure**;

- **strict partial order** (irreflexive, transitive),
- or **preorder** (reflexive, transitive).

Synchronous automata

$\rightsquigarrow$ Define $\prec$ with a *deterministic parity automaton* (DPA) on $V \times V$ that **synchronously** reads two ω-words.

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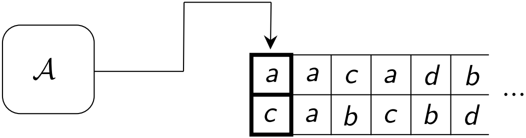

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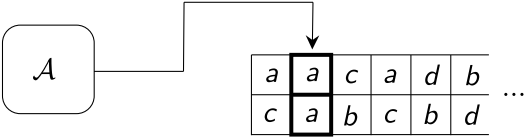

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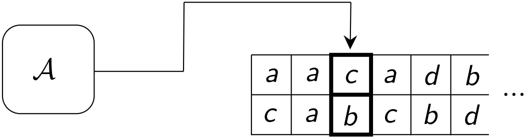

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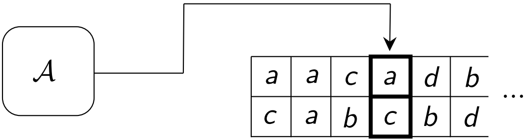

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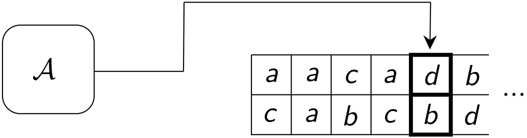

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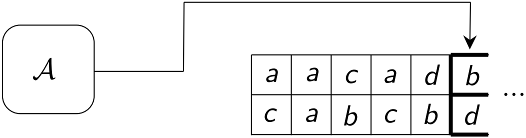

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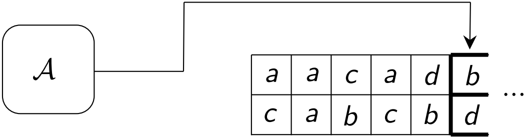

$\mathcal{L}(\mathcal{A}) \subseteq V^\omega \times V^\omega$ can be seen as a binary relation: **ω-Automatic Relation**!

Properties of ω-automatic preference relations

Can we check whether $\prec$ is a strict partial order?

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Deciding whether an ω-automatic relation $\prec$ defined by a DPA is reflexive (resp. irreflexive, transitive, total) is NLOGSPACE-complete.

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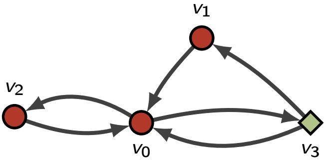
Theorem [B. R. Hodgson, Décidabilité par automate fini, 1983]

The First-Order theory of every ω-automatic structure is decidable.

Games with ω-automatic preference relations

A **game** is a tuple $\mathcal{G} = (A, (\prec_i)_{i \in \mathcal{P}})$ with **ω-automatic strict partial orders** for the players.



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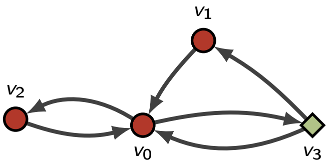

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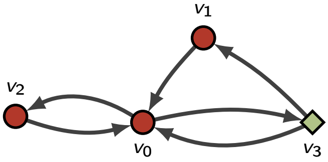

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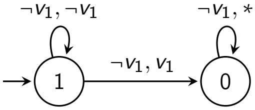

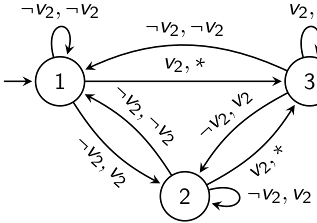

Nash equilibria and decisions problems

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Decision problems for games with ω-automatic strict partial orders:

- (**NE existence**) Does there exist an NE σ?
- (**Constrained NE existence**) Given some threshold lassoes $(\rho_i)_{i \in \mathcal{P}}$, does there exist an NE σ such that $\rho_i <_i \langle \sigma \rangle_{v_0}$?
- (**NE checking**) Given finite-memory strategies $(\sigma_i)_{i \in \mathcal{P}}$, do they form an NE?
- (**NE outcome checking**) Given a lasso π, is it an NE outcome?

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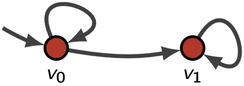

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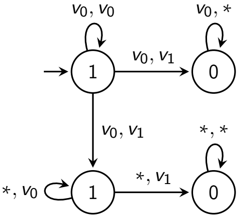

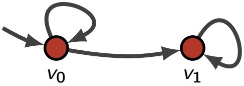

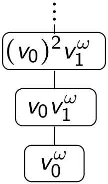

Results

	NE Existence	Constrained NE Existence
No restriction	2EXPTIME	2EXPTIME
$\lvert\mathcal{P}\rvert$ fixed	EXPTIME	-
$\lvert\mathcal{P}\rvert$ and d_i fixed[2]	-	EXPTIME
$\lvert\mathcal{P}\rvert = 1$	PSPACE-complete	PSPACE-complete

In case of existence: **finite-memory** strategies!

[2] Parity acceptance max range

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In case of existence: **finite-memory** strategies!

Theorem (NE (Outcome) Checking)

- The NE checking problem is PSPACE-complete.
- The NE outcome checking problem is in NP $\cap$ coNP and Parity-hard.

[2] Parity acceptance max range

Solve the NE Existence Problem

Solve a zero-sum game.[3]

[3] Bruyère, Raskin, Reynouard, Van Den Bogaard, *The Non-Cooperative Rational Synthesis Problem for Subgame Perfect Equilibria and omega-regular Objectives*, CONCUR 2025.

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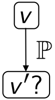

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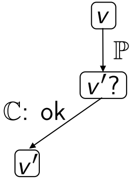

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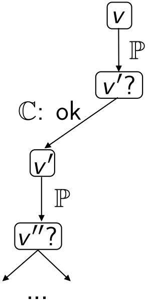

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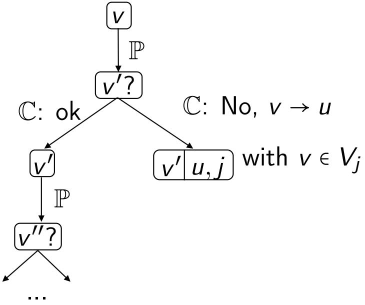

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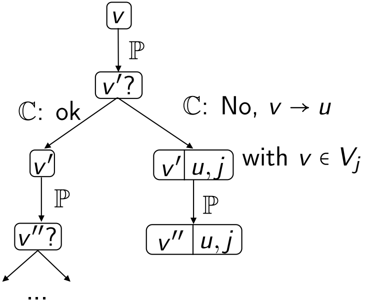

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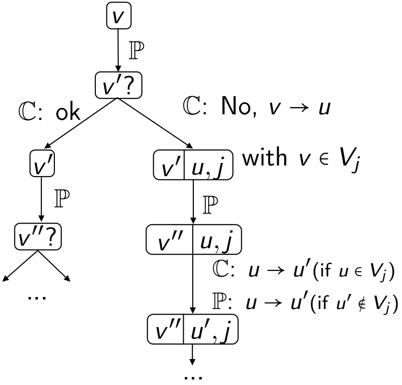

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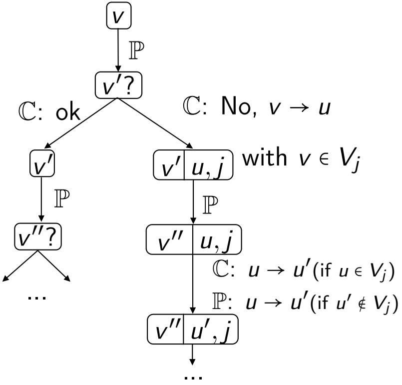

$\exists \sigma_{\mathbb{P}_{\text{“left”}}} \; \forall \sigma_{\mathbb{C}} \; \exists \sigma_{\mathbb{P}_{\text{“right”}}}, \quad \mathbb{P}$ wins.

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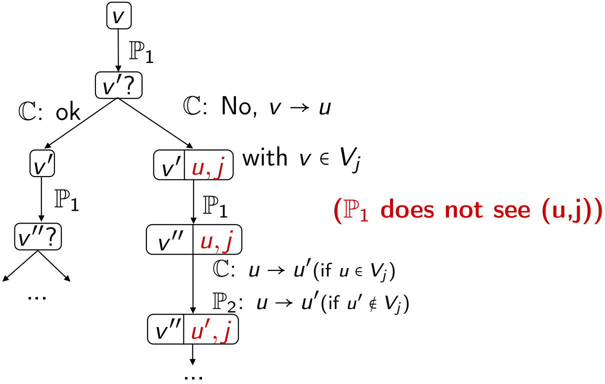
Solve the NE Existence Problem

Solve a **three-player** "zero-sum" game with **imperfect information**.[3]



$$\exists \sigma_{\mathbb{P}_1} \ \forall \sigma_{\mathbb{C}} \ \exists \sigma_{\mathbb{P}_2}, \quad \mathbb{P}_1 \text{ and } \mathbb{P}_2 \text{ win}$$

$\mathbb{P}_1 \mathbb{C} \mathbb{P}_2$ game with imperfect information on $\mathbb{P}_1$!

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ω-Recognizable Relations

A relation $\prec$ is **ω-recognizable** if:

$$\exists k \in \mathbb{N}, \quad \prec = \bigcup_{i=1}^{k} L_i \times R_i \qquad \text{(for } L_i \text{ and } R_i\ \omega\text{-regular)}$$

[4] Bergsträßer, Ganardi, *Revisiting Membership Problems in Subclasses of Rational Relations*, LICS 2023

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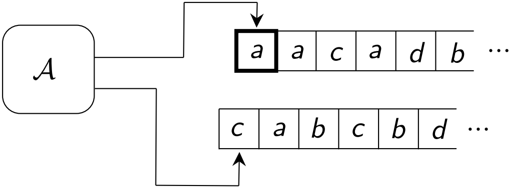

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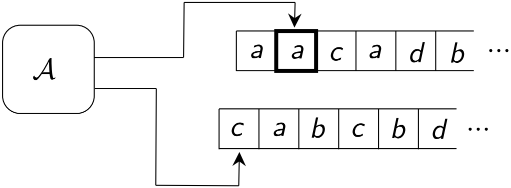

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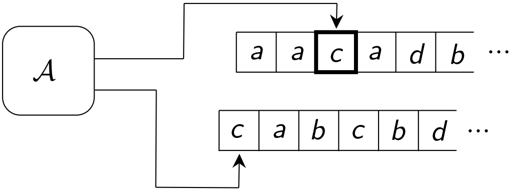

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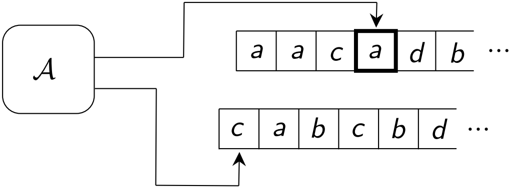

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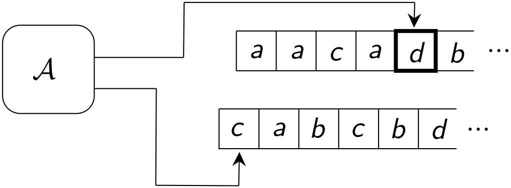

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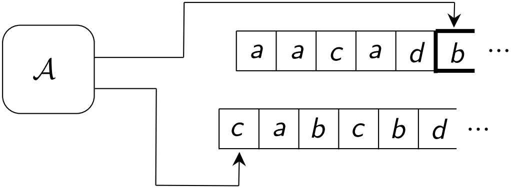

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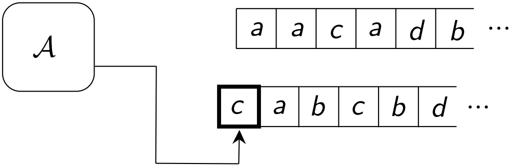

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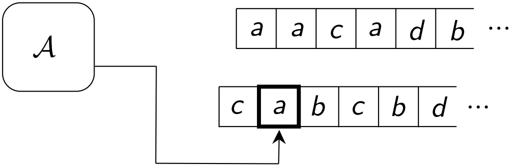

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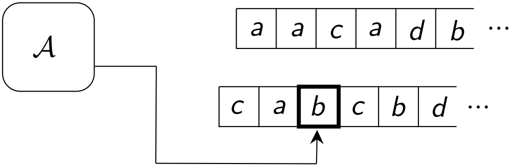

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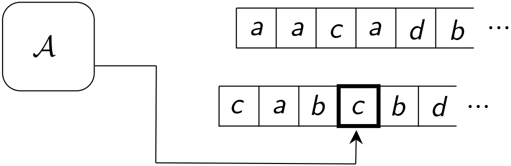

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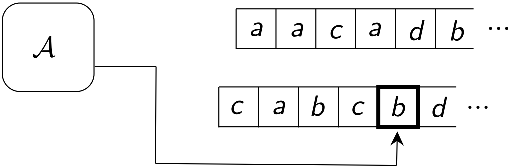

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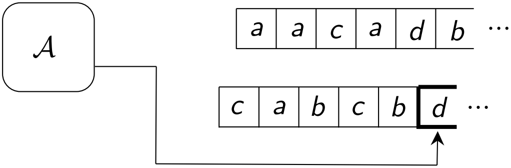

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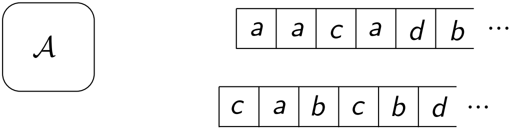

ω-Recognizable Relations $\subsetneq$ ω-Automatic Relations

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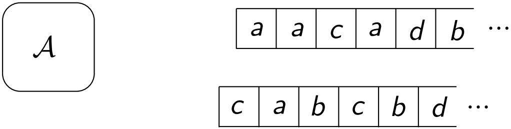

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NLOGSPACE-**complete**[4]

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When $\precsim$ is a preorder, x and y are **equivalent**, $x \sim y$, if $x \precsim y$ and $y \precsim x$.

Corollary of [Löding, Spinrath, 2019]

An ω-automatic preorder $\precsim$ is ω-recognizable if and only if its equivalence relation $\sim$ has a finite index.

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There exists a Nash Equilibrium in every game with ω-recognizable preorders (resp. strict weak orders).

Follows ideas of [Ummels, *The Complexity of Nash Equilibria in Infinite Multiplayer Games*, 2008].

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Thank you! Questions?