Games with ω-Automatic Preference Relations

CFV

Christophe Grandmont
Joint work with Véronique Bruyère and Jean-François Raskin

UMONS Université de Mons

ULB Université Libre de Bruxelles

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$\rightsquigarrow$ Study this rationality, ensure some specification under rational assumptions.

Game played on graphs

$\rightsquigarrow$ How to model these interactions?

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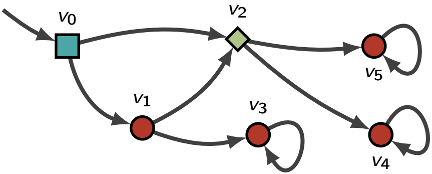
$\rightsquigarrow$ Model interactions with **games played on graphs**.



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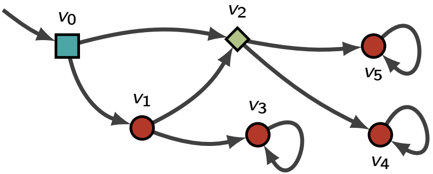

$$\left.\begin{array}{l}\text{Directed graph: } (V, E) \\ \text{Set of players: } \mathcal{P} = \{1, \ldots, n\} \\ \text{Partition of } V\text{: } (V_i)_{i \in \mathcal{P}}\end{array}\right\} \text{Arena } A = (V, E, \mathcal{P}, (V_i)_{\in \mathcal{P}})$$

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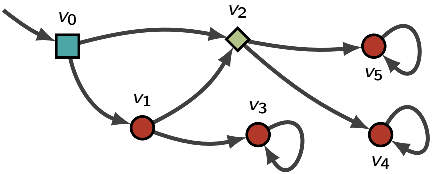

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- Play: $\pi \in \text{Plays} \subseteq V^\omega$ consistent with E, history: $h \in V^*$,
- Strategy for $i \in \mathcal{P}$: function $\sigma_i : V^* V_i \to V$, $hv \mapsto \sigma_i(hv)$.

Example - Games played on graphs

Define the following goals:

- player $\circ$ wants to visit v_1 at least once,
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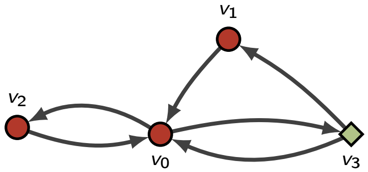

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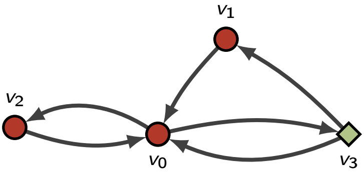

Can players ensure their goals?

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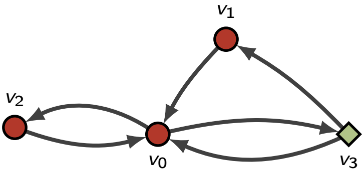


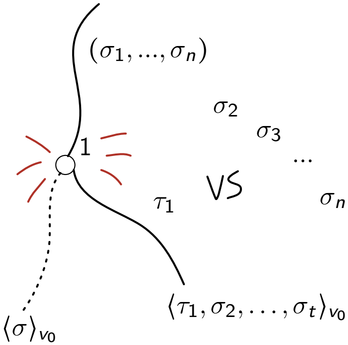
Can players ensure their goals?

No.

Nash Equilibria

A **Nash Equilibrium** (NE) is a strategy profile $\sigma = (\sigma_i)_{i \in \mathcal{P}}$ from which no player has the incentive to **unilaterally deviate**, i.e.,

$$\forall i \in \mathcal{P}, \forall \tau_i \text{ stategy of player } i,\ \langle \sigma_{-i}, \tau_i \rangle_{v_0} \text{ is not better than } \langle \sigma \rangle_{v_0} \textbf{ for } i.$$



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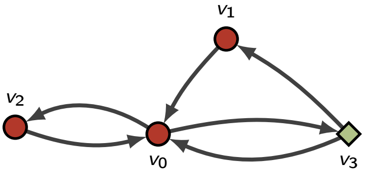

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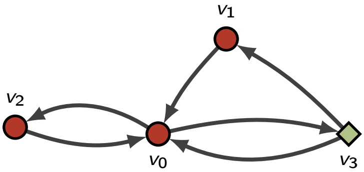

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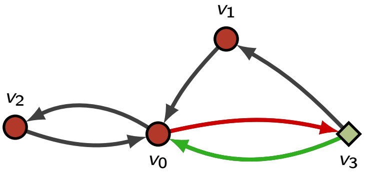

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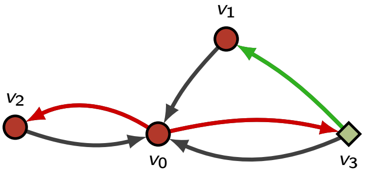

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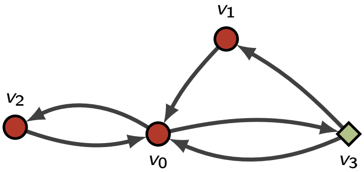

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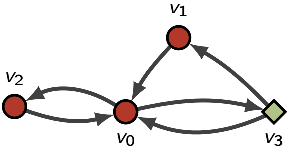
Note: $\langle \sigma' \rangle_{v_0}$ is better than $\langle \sigma \rangle_{v_0}$ for both players.

Broader objectives

Classical setting: **objectives** $\Omega_i : \text{Plays} \to \mathbb{Q}$.



What if we take more complex objectives for both players?

Study NEs in all cases?

[1]See, e.g., Bouyer et al., *Nash Equilibria for Reachability Objectives in Multi-player Timed Games*, or Pauly, Le Roux, *Equilibria in multi-player multi-outcome infinite sequential games.*

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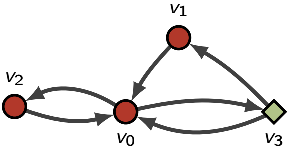

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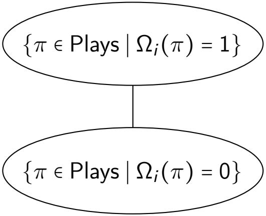

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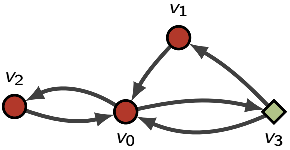

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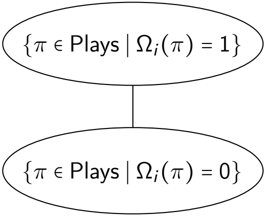

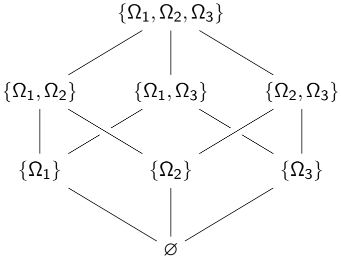

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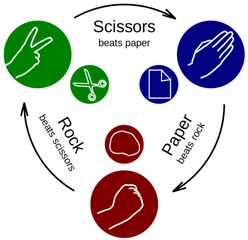
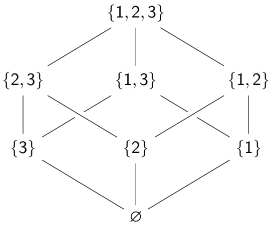
Properties of preference relations





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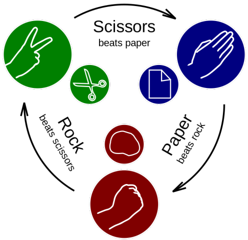

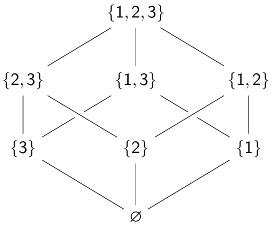

Usual properties

- Reflexivity: $\forall x,\ x \prec x$,
- Irreflexivity: $\forall x,\ x \not\prec x$,
- Transitivity: $\forall x, y, z,\ x \prec y \wedge y \prec z \Rightarrow x \prec z$,
- Totality: $\forall x, y,\ x \neq y \Rightarrow x \prec y \vee y \prec x$,
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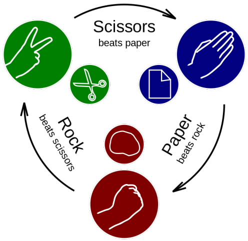

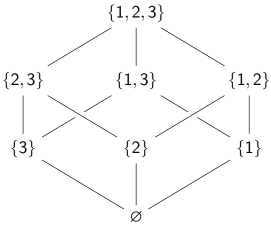

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We expect $\prec$ to have an **order structure**;

- **strict partial order** (irreflexive, transitive),
- or **preorder** (reflexive, transitive).

Synchronous automata

$\rightsquigarrow$ Define $\prec$ with a *deterministic parity automaton* (DPA) on $V \times V$ that **synchronously** reads two ω-words.

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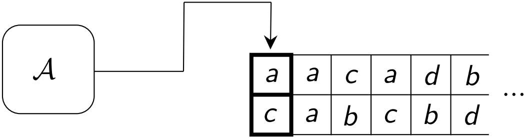

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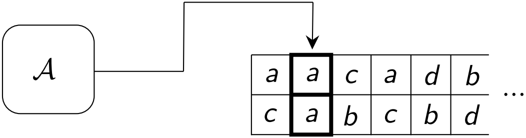

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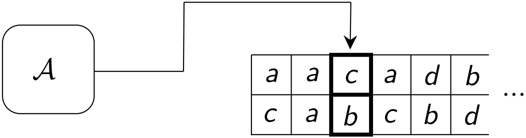

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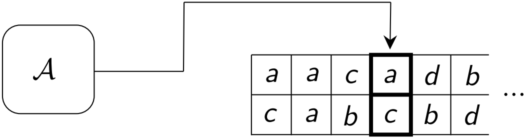

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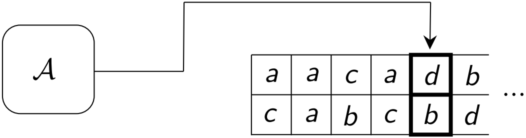

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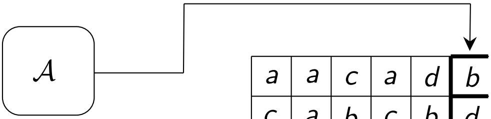

a	a	c	a	d	b	$\dots$
c	a	b	c	b	d	

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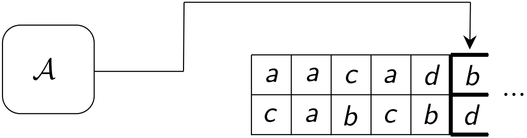

$\mathcal{L}(\mathcal{A}) \subseteq V^\omega \times V^\omega$ can be seen as a binary relation: **ω-Automatic Relation**!

Properties of ω-automatic preference relations

Can we check whether $\prec$ is a strict partial order?

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Proposition

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Theorem [B. R. Hodgson, Décidabilité par automate fini, 1983]

The First-Order theory of every ω-automatic structure is decidable.

Games with ω-automatic preference relations

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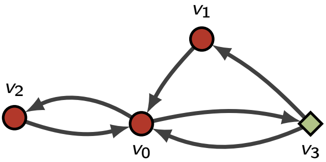

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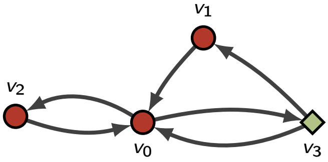

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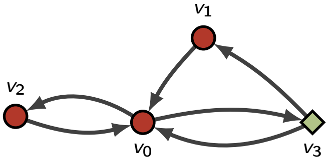

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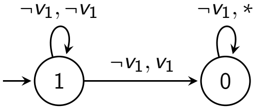

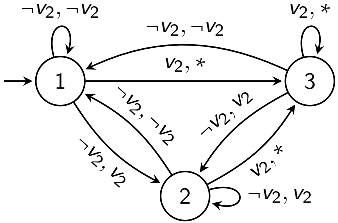

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Decision problems for games with ω-automatic strict partial orders:

- (**NE existence**) Does there exist an NE σ?
- (**Constrained NE existence**) Given some threshold lassoes $(\rho_i)_{i \in \mathcal{P}}$, does there exist an NE σ such that $\rho_i <_i \langle \sigma \rangle_{v_0}$?
- (**NE checking**) Given finite memory strategies $(\sigma_i)_{i \in \mathcal{P}}$, do these form an NE?
- (**NE outcome checking**) Given a lasso π, is it an NE outcome?

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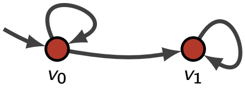

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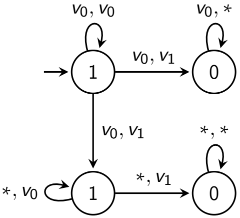

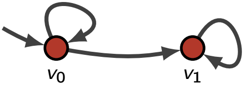

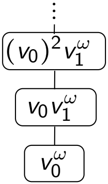

Results

	NE Existence	Constrained NE Existence
No restriction	2EXPTIME	2EXPTIME
$\lvert\mathcal{P}\rvert$ fixed	EXPTIME	-
$\lvert\mathcal{P}\rvert$ and d_i fixed[2]	-	EXPTIME
$\lvert\mathcal{P}\rvert = 1$	PSPACE-complete	PSPACE-complete

In case of existence: **finite-memory** strategies!

[2] Parity acceptance max range

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In case of existence: **finite-memory** strategies!

Theorem (NE (Outcome) Checking)

- The NE checking problem is PSPACE-complete.
- The NE outcome checking problem is in NP $\cap$ coNP and Parity-hard.

[2] Parity acceptance max range

Solve a three-player "zero-sum" game with **imperfect information**.[3]

[3] Bruyère, Raskin, Reynouard, Van Den Bogaard, *The Non-Cooperative Rational Synthesis Problem for Subgame Perfect Equilibria and omega-regular Objectives.*

Algorithm for the NE Existence Problem

Solve a three-player "zero-sum" game with **imperfect information**.[3]

- $\mathbb{PC}$ game: $\mathbb{P}$ creates π; $\mathbb{C}$ can deviate at any time at $v \in V_i$; if so, $\mathbb{P}$ retaliates to show that the deviation π' is such that $\pi \not<_i \pi'$.

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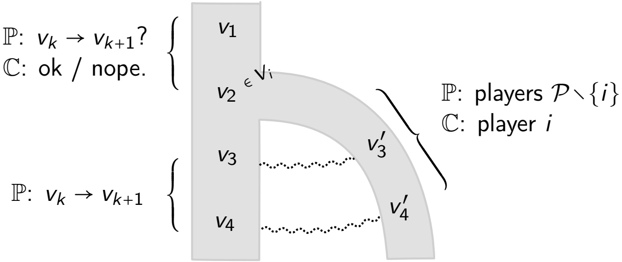

$$\exists \sigma_{\mathbb{P}_{\text{"left"}}} \ \forall \sigma_{\mathbb{C}} \ \exists \sigma_{\mathbb{P}_{\text{"right"}}}, \quad \mathbb{P} \text{ wins.}$$

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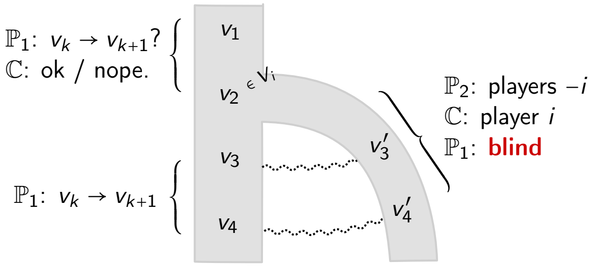
Algorithm for the NE Existence Problem

Solve a three-player "zero-sum" game with **imperfect information**.[3]

- $\mathbb{P}_1\mathbb{C}\mathbb{P}_2$ game: $\mathbb{P}_1$ creates π; $\mathbb{C}$ can deviate at any time at $v \in V_i$; if so, $\mathbb{P}_2$ retaliates to show that the deviation π' is such that $\pi \not<_i \pi'$.



$$\exists \sigma_{\mathbb{P}_1} \ \forall \sigma_{\mathbb{C}} \ \exists \sigma_{\mathbb{P}_2}, \quad \mathbb{P}_1 \text{ and } \mathbb{P}_2 \text{ win}$$

$\mathbb{P}_1\mathbb{C}\mathbb{P}_2$ game with imperfect information on $\mathbb{P}_1$!

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ω-Recognizable Relations

A relation $\prec$ is **ω-recognizable** if there exists $k \in \mathbb{N}$ such that $\prec = \bigcup_{i=1}^{k} L_i \times R_i$, for $L_i, R_i \subseteq V^\omega$, ω-regular languages.

[4] Löding, Spinrath, *Decision Problems for Subclasses of Rational Relations over Finite and Infinite Words*, 2019.

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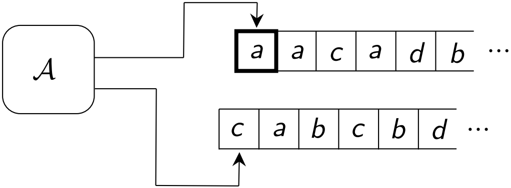

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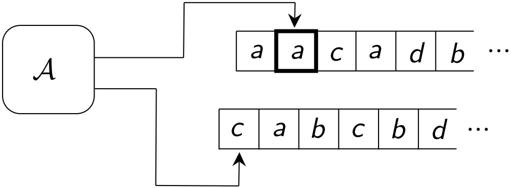

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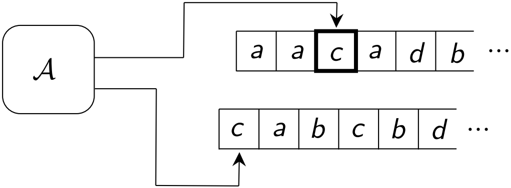

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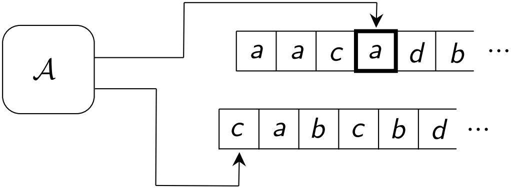

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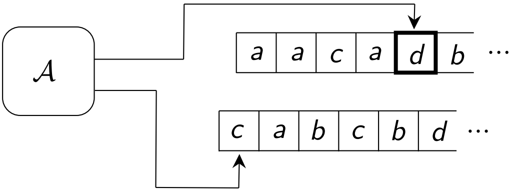

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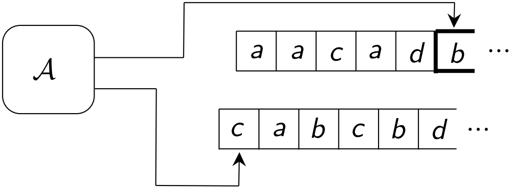

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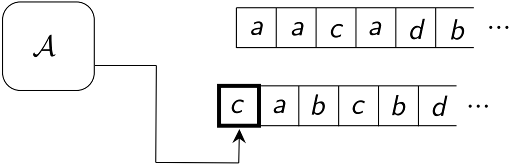

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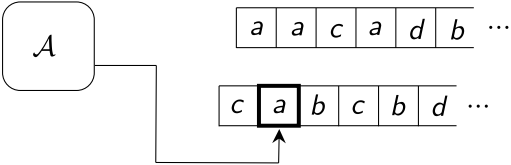

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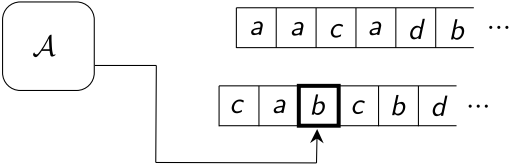

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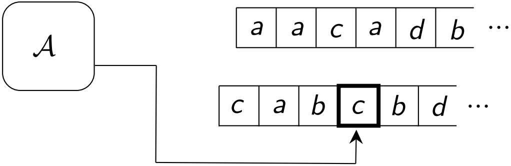

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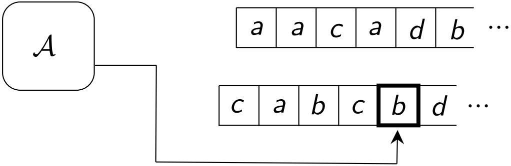

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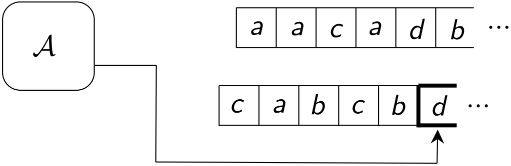

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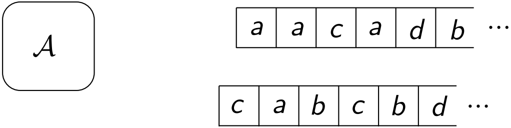

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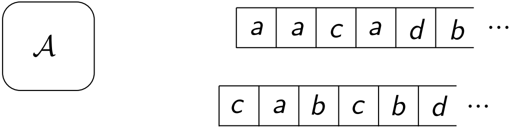

ω-Recognizable Relations $\subsetneq$ ω-Automatic Relations.

We can decide in 2EXPTIME whether an ω-automatic relation is ω-recognizable[4]

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ω-Recognizable Preorder

When $\precsim$ is a preorder, x and y are **equivalent**, $x \sim y$, if $x \precsim y$ and $y \precsim x$.

Corollary of [Löding, Spinrath, 2019]

An ω-automatic preorder $\precsim$ is ω-recognizable if and only if its equivalence relation $\sim$ has a finite number of equivalence classes.

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Follows ideas of [Ummels, *The Complexity of Nash Equilibria in Infinite Multiplayer Games*, 2008].

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Thank you! Questions?