As Soon as Possible but Rationally: Rational Synthesis for Reachability on Weighted Graphs

Highlights'24

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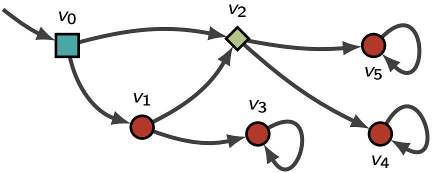
Multiplayer Reachability Games

$\rightsquigarrow$ Model interactions with (turn-based) **games played on graphs**



- Player 0: **system**, players $1, \ldots, t$: **component of the environment**...

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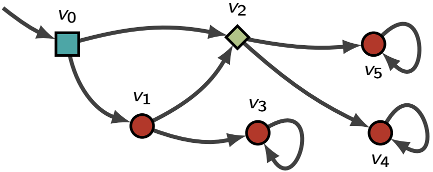

- Player 0: **system**, players $1, \ldots, t$: **component of the environment**...
- ...each with a **quantitative** reachability objective (T_i, w_i):
  - $T_i \subseteq V$ a **target set**,
  - $w_i : E \to \mathbb{N}$ a **weight function**,

⇝ associate a **cost** to each play $\pi = \pi_0\pi_1\pi_2 \ldots$,

$$\mathrm{cost}_i(\pi) = \begin{cases} \sum_{k=1}^{n} w_i((\pi_{k-1}, \pi_k)) & \text{for } n = \inf\{k \in \mathbb{N} \mid \pi_k \in T_i\} < +\infty, \\ +\infty & \text{if } n = +\infty. \end{cases}$$

Rationality in Stackelberg Game

Stackelberg game: player 0 fixes a strategy σ_0, and we expect the environment to behave **rationally** according to σ_0.

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Rationality is … or

- **Nash Equilibrium** (NE): players $1, \ldots, t$ agree for strategies where no one has an incentive to unilaterally deviate.
- **Pareto Optimality**: minimize $(\text{cost}_1, \ldots, \text{cost}_t)$.

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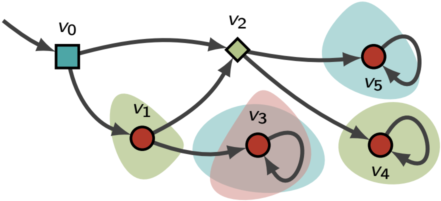
A strategy profile $\sigma = (\sigma_0, \ldots, \sigma_t)$ is an NE if for each player i $(\neq 0)$, each **strategy** τ_i of i (called **deviating**),

$$\mathsf{cost}_i(\langle \sigma \rangle) \leq \mathsf{cost}_i(\langle \sigma_0, \ldots, \sigma_{i-1}, \tau_i, \sigma_{i+1}, \ldots, \sigma_t \rangle).$$

The play $\langle \sigma \rangle$ is called an **NE outcome**.

Nash Equilibria in Stackelberg Game: Example



- All weights constant, $w_i = 1$,
- $T_0 = \{v_3\}$,
- $T_\square = \{v_3, v_5\}$,
- $T_\diamond = \{v_1, v_4\}$.

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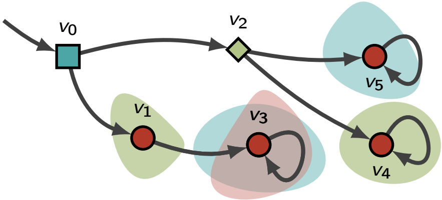

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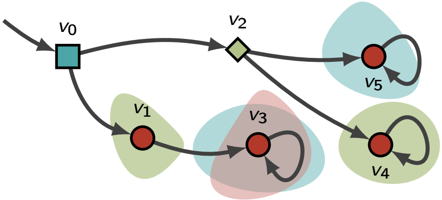

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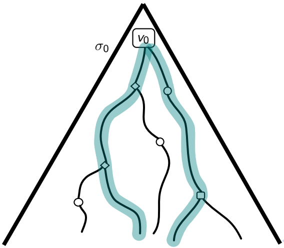
For σ_0 such that $\sigma_0(v_1) = v_3$:

	player 0	player $\square$	player $\diamond$	NE outcome ?
$v_0 v_1 v_3$	2	2	1	Yes
$v_0 v_2 v_5$	$+\infty$	2	$+\infty$	No (player $\diamond$: $v_2 \to v_4$)
$v_0 v_2 v_4$	$+\infty$	$+\infty$	2	No (player $\square$: $v_0 \to v_1$)

Rational Synthesis

The (non-cooperative) **Rational Synthesis** problem asks to decide whether there exists a strategy of player 0 such that every *rational* response of the environment *to this strategy* produces a play π with $\text{cost}_0(\pi) \leq c$, i.e.,

$$\exists \sigma_0, \ \forall \pi \in \text{Plays}_{\sigma_0}, \ \ \pi \text{ is an NE outcome} \Rightarrow \text{cost}_0(\pi) \leq c.$$



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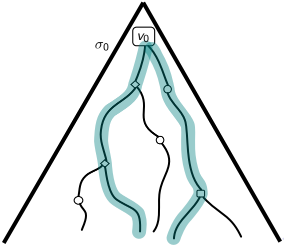
$\rightsquigarrow$ Less restricted version: **Cooperative** Rational Synthesis:

$$\exists \sigma_0, \ \exists \pi \in \mathsf{Plays}_{\sigma_0}, \ \ \pi \text{ is an NE outcome} \wedge \mathsf{cost}_0(\pi) \leq c.$$



All results

Verification: give σ_0 as input through a nondeterministic or deterministic **Mealy machine**

[1] NEXPTIME-complete for a *common weight function*: Brihaye, Bruyère, and Reghem, "Quantitative Reachability Stackelberg-Pareto Synthesis is NEXPTIME-complete", RP 2023.

[2] Bruyère, Raskin, and Tamines, "Stackelberg-Pareto Synthesis", CONCUR 2021

[3] Condurache et al., "The Complexity of Rational Synthesis", ICALP 2016

[4] Christophe Grandmont, Master's Thesis 2023

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	Coop. synthesis	Non-coop. synthesis	Non-coop. verif. (det.)	Non-coop. verif. (nondet.)
Pareto, weights	PSPACE-**complete**	NEXPTIME-hard[1]	Π_2^P-**complete**	PSPACE-**complete**
Pareto, qualitative	PSPACE-**complete**	NEXPTIME-complete[2]	Π_2^P-**complete**	PSPACE-**complete**
Nash, weights	NP-**complete**	Unknown (EXPTIME-**hard**)	coNP-**complete**	coNP-**complete**
Nash, qualitative	NP-complete[3]	PSPACE-complete[3]	coNP-**complete**[4]	coNP-**complete**[4]
Pareto, weights in $\mathbb{Z}$	-	**Undecidable**	-	-
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