As Soon as Possible but Rationally

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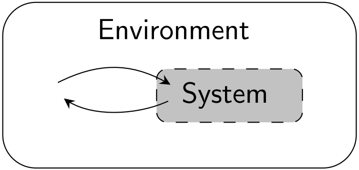
Work with Véronique Bruyère and Jean-François Raskin (Available on arXiv!)

Topic

A **system** (player 0) interacting with the **environment** (player 1).



- System is **controllable** (coffee machine, elevator, autopilot, ...)
- Environment is not (humans, other independent systems, ...)

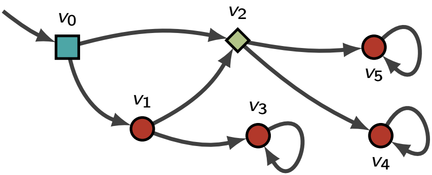
$\rightsquigarrow$ How to guarantee a **specification** Ω_0 in such reactive system ?

Topic

Guarantee Ω_0, in which condition ?

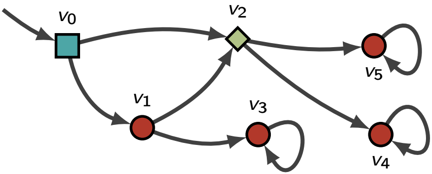
- **Zero-sum** ? Environment completely antagonistic... $\rightsquigarrow$ Too simple !
- Instead, assume that the environment is composed of **multiple components** $1, \ldots, k$, each with a specification Ω_i.

$\rightsquigarrow$ Model these interactions with **games played on graphs**...



... where the environment **plays "rationally"** !

Game played on graphs: Arena



$$\left.\begin{array}{l}\text{Directed graph: } (V, E)\\ \text{Set of players: } \mathcal{P} = \{0, \ldots, k\}\\ \text{Partition of } V\text{: } (V_i)_{i\in\mathcal{P}}\end{array}\right\}\text{Arena } \mathcal{A} = (V, E, \mathcal{P}, (V_i)_{\in\mathcal{P}})$$

- Play: $\pi \in V^\omega$ consistent with E, history: $h \in V^*$ consistent with E,
- Strategy for $i \in \mathcal{P}$: function $\sigma_i : V^* V_i \to V$.

Game played on graphs: Reachability objective

Associate an objective to each player:

$\rightsquigarrow$ **Reachability** (T_i, w_i) where

- $T_i \subseteq V$, $w_i : E \to \mathbb{N}$,
- for a play $\pi = \pi_0\pi_1 \dots$ and $n = \inf\{k \in \mathbb{N} \mid \pi_k \in T_i\}$,

$$\mathrm{cost}_i(\pi) = \begin{cases} \Sigma_{k=1}^{n} w_i((\pi_{k-1}, \pi_k)) & \text{if } n < +\infty, \\ +\infty & \text{otherwise.} \end{cases}$$

(When $w_i(e) = 0$ for all $e \in E \rightsquigarrow$ **qualitative**)

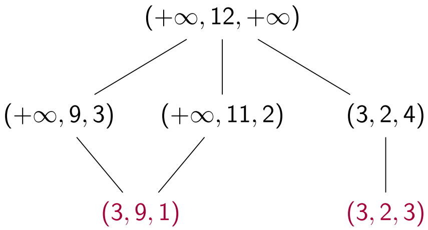
$\rightsquigarrow$ Goal of the system: $\mathrm{cost}_0 \leq c$, for a threshold $c \in \mathbb{N}$.
$\rightsquigarrow$ Goal of each player of the environment: **Minimize** cost_i, then be antagonistic towards the system.

What does "minimize" mean (i.e., be rational) ?

Pareto-Optimality

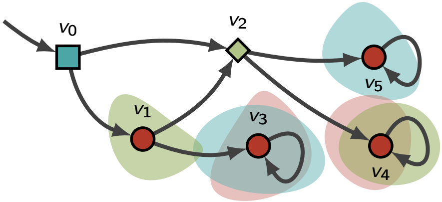
Players $1, \ldots, k$ agree to get ~~the~~ a lowest $\text{cost}_{\text{env}} = (\text{cost}_1, \ldots, \text{cost}_k)$: **Pareto-Optimality** !

$\rightsquigarrow$ Partial order $\preceq$ on $\mathbb{N}^k$, e.g.



For σ_0 fixed, $P_{\sigma_0} = \min\{\text{cost}_{\text{env}}(\pi) \mid \pi \text{ play consistent with } \sigma_0\}$.

Pareto-Optimality: Example



- All weights constant, $w_i = 1$,
- $T_0 = \{v_3, v_4\}$,
- $T_1 = \{v_3, v_5\}$,
- $T_2 = \{v_1, v_4\}$.

For σ_0 such that $\sigma_0(v_1) = v_2$:

$$\left.\begin{array}{llll} v_0 v_1 v_2 v_5 & : \text{player 0:} +\infty, & \text{env:} & (3,1) \\ v_0 v_1 v_2 v_4 & : \text{player 0: } 3, & \text{env:} & (+\infty,1) \\ v_0 v_2 v_5 & : \text{player 0:} +\infty, & \text{env:} & (2,+\infty) \\ v_0 v_2 v_4 & : \text{player 0: } 2, & \text{env:} & (+\infty,2) \end{array}\right\} P_{\sigma_0} = \{(3,1),(2,+\infty)\}$$

For σ_0' such that $\sigma_0'(v_1) = v_3$:

$$\left.\begin{array}{llll} v_0 v_1 v_3 & : \text{player 0: } 2, & \text{env:} & (2,1) \\ v_0 v_2 v_5 & : \text{player 0:} +\infty, & \text{env:} & (2,+\infty) \\ v_0 v_2 v_4 & : \text{player 0: } 2, & \text{env:} & (+\infty,2) \end{array}\right\} P_{\sigma_0} = \{(2,1),(+\infty,2)\}$$

Pareto Synthesis

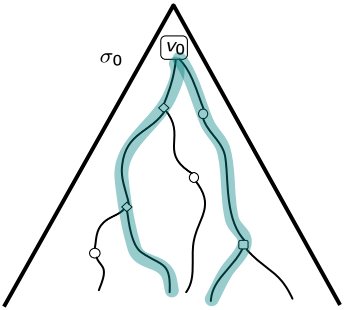
Let $\mathcal{G} = (V, E, V_0, (T_i, w_i)_{0 \leq i \leq k})$ and $c \in \mathbb{N}$,

Non-Cooperative Pareto Synthesis (NCPS) problem:

$$\exists \sigma_0,\ \forall \pi \in \mathsf{Plays}_{\sigma_0},\quad \mathsf{cost}_{\mathsf{env}}(\pi) \in P_{\sigma_0} \Rightarrow \mathsf{cost}_0(\pi) \leq c.$$

Cooperative Pareto Synthesis (CPS) problem:

$$\exists \sigma_0,\ \exists \pi \in \mathsf{Plays}_{\sigma_0},\quad \mathsf{cost}_{\mathsf{env}}(\pi) \in P_{\sigma_0} \wedge \mathsf{cost}_0(\pi) \leq c.$$



Pareto Results

	Coop. synthesis	Non-coop. synthesis
Pareto, weights	PSPACE-**complete**	NEXPTIME-complete[1]
Pareto, qualitative	PSPACE-**complete**	NEXPTIME-complete[2]

Verification variants: given a strategy σ_0 defined by a **deterministic** Mealy machine, is σ_0 a solution to a synthesis problem ? (Is every σ_0 defined by a **nondeterministic** Mealy machine a solution ?)

	Non-coop. verif. (det.)	Non-coop. verif. (nondet.)
Pareto, weights	Π_2^P-**complete**	PSPACE-**complete**
Pareto, qualitative	Π_2^P-**complete**	PSPACE-**complete**

[1] Brihaye, Bruyère, and Reghem, "Quantitative Reachability Stackelberg-Pareto Synthesis is NEXPTIME-Complete", RP 2023

[2] Bruyère, Raskin, and Tamines, "Stackelberg-Pareto Synthesis", CONCUR 2021

All results

Nash variants: instead of asking $\mathrm{cost}_{\mathrm{env}}(\pi) \in P_{\sigma_0}$, ask that π is the outcome of a **Nash Equilibrium**.
Restricted environment: e.g. force $\mathcal{P} = \{0, 1\}$ for a 1-env (one-player).

	Coop. synthesis	Non-coop. synthesis	Non-coop. verif. (det.)	Non-coop. verif. (nondet.)
Pareto, weights	PSPACE-**complete**	NEXPTIME-complete[3]	Π_2^P-**complete**	PSPACE-**complete**
Pareto, qualitative	PSPACE-**complete**	NEXPTIME-complete[4]	Π_2^P-**complete**	PSPACE-**complete**
Nash, weights	NP-**complete**	Unknown (EXPTIME-**hard**)	coNP-**complete**	coNP-**complete**
Nash, qualitative	NP-complete[5]	PSPACE-complete[5]	coNP-**complete**[6]	coNP-**complete**[6]
Nash, weights, 2-env	-	Unknown (EXPTIME-**hard**)	-	-
Nash, weights, 1-env	NP-**complete**	EXPTIME **(**PSPACE-**hard)**	coNP	coNP-**complete**

Thank you !

[3] Brihaye, Bruyère, and Reghem, "Quantitative Reachability Stackelberg-Pareto Synthesis is NEXPTIME-Complete", RP 2023

[4] Bruyère, Raskin, and Tamines, "Stackelberg-Pareto Synthesis", CONCUR 2021

[5] Condurache et al., "The Complexity of Rational Synthesis", ICALP 2016

[6] Christophe Grandmont, Master's Thesis 2023